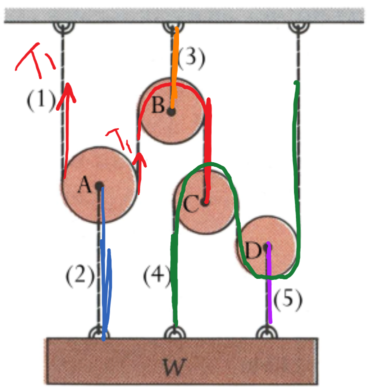
Equilibrium of a system of particles

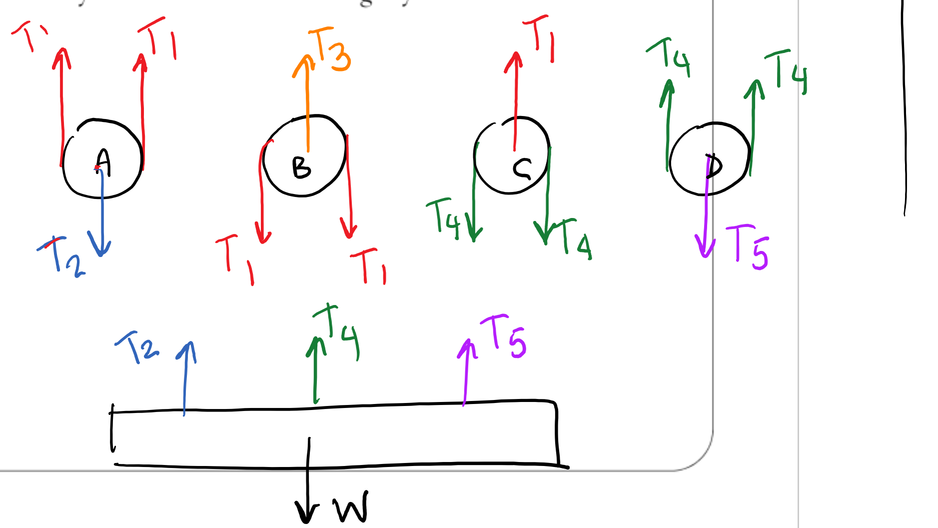
Some practical engineering problems involve the statics of interacting or interconnected particles. To solve them, we use Newton's first law

$$\Sigma \mathbf{F} = \mathbf{0}$$

on selected multiple free-body diagrams of particles or groups of particles.



The five ropes can each take 1500 N without breaking. How heavy can W be without breaking any?



A; $\Sigma F = 0 \Rightarrow 2T_1 - T_2 = 0 \longrightarrow 2T_1 = T_2 \qquad T_2 > T_1$

B: $T_3 - 2T_1 = 0 \longrightarrow 2T_1 = T_3 \longrightarrow T_3 > T_1 \qquad T_3 = T_2$

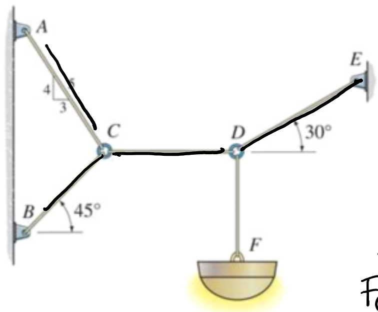
C: $T_1 - 2T_4 = 0 \longrightarrow T_1 = 2T_4 \longrightarrow T_4 = \frac{T_1}{2} = \frac{T_2}{4}$

D: $2T_4 - T_5 = 0 \longrightarrow T_5 = 2T_4 = \frac{T_2}{2}$

Block: $T_2 + T_4 + T_5 - W = 0 \longrightarrow T_2 + \frac{T_2}{4} + \frac{T_2}{2} = W$

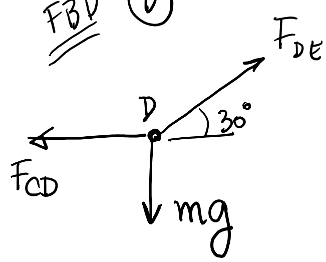
$$(4+1+2)\frac{T_2}{4} = W$$

$T_2 = \frac{4W}{7} \qquad W = \frac{7T_2}{4} = \frac{7 \; 1500\,N}{4} = 2.63\,kN$



Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400N.

FBD (D)



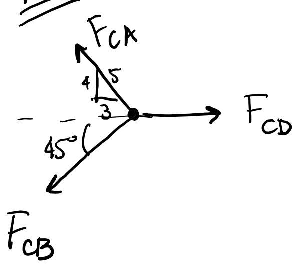
$\sum F_y = 0 \therefore F_{DE}\sin 30° - mg = 0$

$F_{DE} = \dfrac{mg}{\sin 30°} = \underline{19.62}\ m$ //

$\sum F_x = 0 \therefore F_{DE}\cos 30° - F_{CD} = 0$

$F_{CD} = \underline{16.99}\ m$ //

FBD (C)



$\sum F_x = 0 \therefore F_{CD} - F_{CA}\dfrac{3}{5} - F_{CB}\cos 45° = 0$

$\sum F_y = 0 \therefore F_{CA}\dfrac{4}{5} - F_{CB}\sin 45° = 0$

$F_{CA} = F_{CB}\sin 45°\left(\dfrac{5}{4}\right)$

$F_{CD} - F_{CB}\sin 45°\left(\dfrac{5}{4}\right)\left(\dfrac{3}{5}\right) - F_{CB}\cos 45° = 0$

$F_{CD} = 16.99\ m$

$F_{CB} = \underline{13.73}\ m$ //

$F_{CA} = \underline{12.14}\ m$ //

$F_{DE} = 19.62\ m < 400\ N \Rightarrow \boxed{m \leq 20.4\ kg}$

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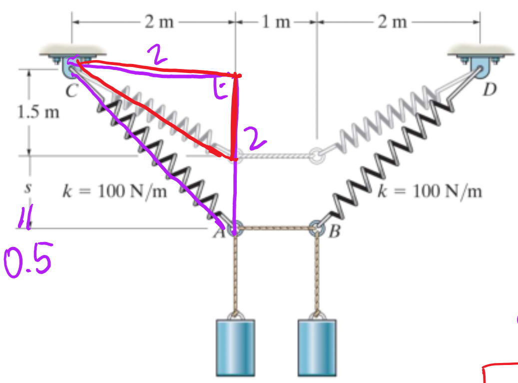
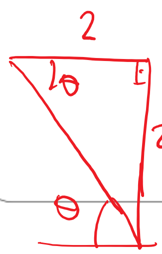


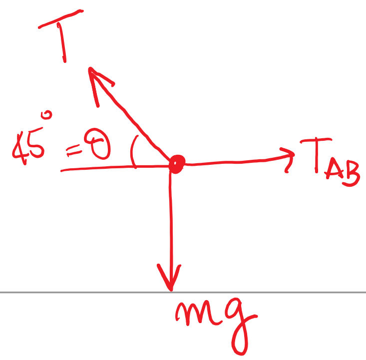
Determine the mass of each of the two identical cylinders if they cause a sag of $\boxed{s = 0.5\ m}$ when suspended from the rings at A and B. Note that s is zero when the cylinders are removed.

$$T = k\delta = 100\ N/m\ \delta = ?$$

$$\delta = L_{final} - L_{initial} = \sqrt{2^2 + 2^2} - \sqrt{2^2 + 1.5^2}$$

$$\boxed{T = 32.8\ N}$$

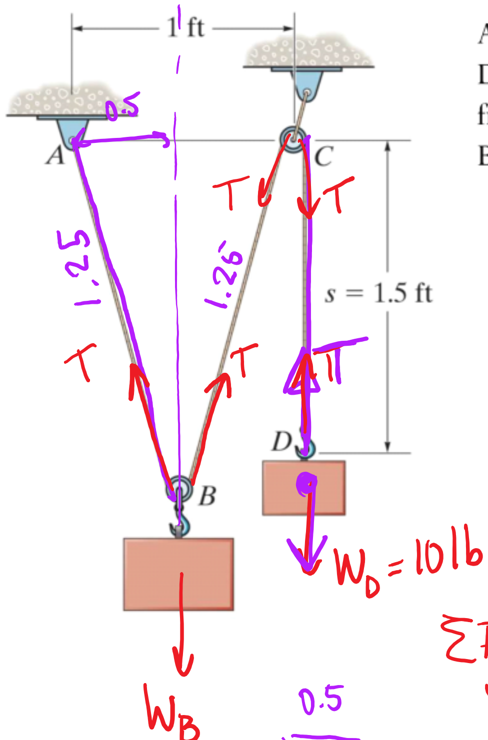




$$\Sigma F_x = 0 \quad \therefore \quad T\cos 45^\circ = T_{AB}$$

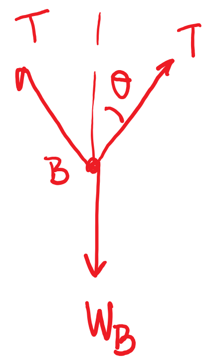
$$\Sigma F_y = 0 \quad \therefore \quad T\sin 45^\circ = mg$$

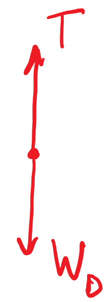
$$\boxed{m = \frac{T\sin 45^\circ}{g}} = 2.37\ Kg$$



L=

A "scale" is constructed with a 4-ft-long cord and the 10-lb block D. The cord is fixed to a pin at A and passes over two *small* frictionless pulleys. Determine the weight of the suspended block B if the system is in equilibrium when s = 1.5ft.



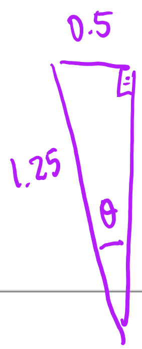


$$\boxed{T = W_D = 10 lb}$$

$$\Sigma F_y = 0:\; 2T\cos\theta - W_B = 0$$

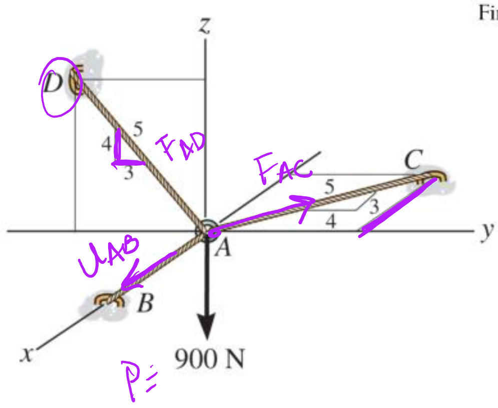
$$\boxed{W_B = 2T\cos\theta}$$

$$\boxed{W_B = 18.3 \text{ lb}}$$



$$\sin\theta = \frac{0.5}{1.25} \Rightarrow \theta = 13.578°$$

3D force systems



Find the tension developed in each cable

$$\vec{F}_{AC} + \vec{F}_{AD} + \vec{F}_{AB} + \vec{P} = 0$$

$$\vec{F}_{AC} = \langle -F_{AC}\tfrac{3}{5},\ F_{AC}\tfrac{4}{5},\ 0 \rangle$$

$$\vec{F}_{AB} = F_{AB}\langle 1, 0, 0 \rangle$$

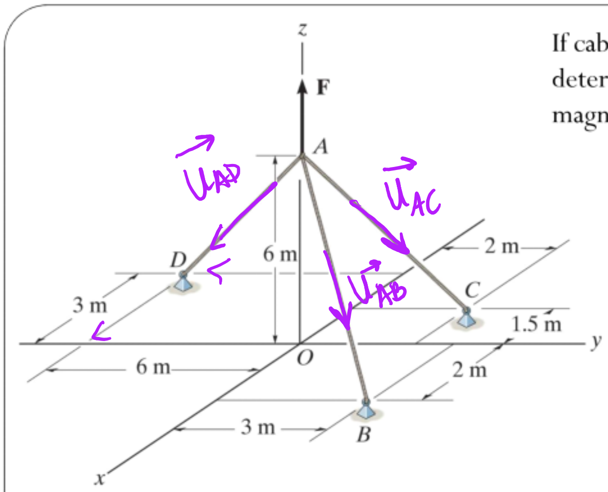
$$\vec{F}_{AD} = F_{AD}\langle 0, -\tfrac{3}{5}, \tfrac{4}{5} \rangle$$

$$\vec{P} = P\langle 0, 0, -1 \rangle$$

$$\Sigma F_x = 0 \therefore -\tfrac{3}{5}F_{AC} + F_{AB} = 0 \longrightarrow F_{AB} = \tfrac{3}{5}F_{AC} = \tfrac{3}{5}(844) = 506\text{ N}$$

$$\Sigma F_y = 0 \therefore F_{AC}\tfrac{4}{5} - \tfrac{3}{5}F_{AD} = 0 \longrightarrow F_{AC} = \tfrac{3}{5}F_{AD}\tfrac{5}{4} = \tfrac{3}{4}(1125) = 844\text{N}$$

$$\Sigma F_z = 0 \therefore \tfrac{4}{5}F_{AD} - P = 0 \longrightarrow F_{AD} = \frac{5P}{4} = 1125\text{ N}$$



If cable AB is subjected to a tension of 700 N, determine the tension in cables AC and AD and the magnitude of the vector **F**

$$\sum \vec{F} = 0$$

$$\vec{r}_A = \langle 0, 0, 6 \rangle \text{ m}$$

$$\vec{r}_B = \langle 2, 3, 0 \rangle \text{ m}$$

$$\vec{r}_C = \langle -1.5, 2, 0 \rangle \text{ m}$$

$$\vec{r}_D = \langle -3, -6, 0 \rangle \text{ m}$$

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A \longrightarrow \vec{u}_{AB} = \vec{r}_{AB} / |\vec{r}_{AB}|$$

$$\vec{r}_{AC} = \vec{r}_C - \vec{r}_A \longrightarrow \vec{u}_{AC} = \vec{r}_{AC} / |\vec{r}_{AC}|$$

$$\vec{r}_{AD} = \vec{r}_D - \vec{r}_A \longrightarrow \vec{u}_{AD} = \vec{r}_{AD} / |\vec{r}_{AD}|$$

$$\vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AD} + F\langle 0, 0, 1 \rangle = 0$$

$$\underbrace{F_{AB}}_{=700\text{N}} \vec{u}_{AB} + F_{AC}\vec{u}_{AC} + F_{AD}\vec{u}_{AD} + F\langle 0, 0, 1 \rangle = 0$$

→ 3 equations, (F_{AC}, F_{AD}, F)